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Analytical Solution for Interface Flow to a Sink With an Upconed Saline Water Lens: Strack's Regimes Revisited

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Abstract

© 2018. American Geophysical Union. All Rights Reserved. A study is made of a steady, two-dimensional groundwater flow with a horizontal well (drain), which pumps out freshwater from an aquifer sandwiched between a horizontal bedrock and ponded soil surface, and containing a lens-shaped static volume of a heavier saline water (DNAPL-dense nonaqueous phase liquid) as a free surface. For flow toward a line sink, an explicit analytical solution is obtained by a conformal mapping of the hexagon in the complex potential plane onto a reference plane and the Keldysh-Sedov integral representation of a mixed boundary-value problem for a complex physical coordinate. The interface is found as a function of the pumping rate, the well locus, the ratio of liquid densities, and the hydraulic heads at the soil surface and in the well. The shape with two inflexion points and fronts varies from a small-thickness bedrock-spread pancake to a critical curvilinear triangle, which cusps toward the sink. The problem is mathematically solvable in a relatively narrow band of geometric and hydraulic parameters. A similar analytic solution for a static heavy bubble confined by a closed-curve interface (no contact with the bedrock) is outlined as an illustration of the method to solve a mixed boundary-value problem.

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Keywords

analytic functions, complex potential, Keldysh-Sedov formulae, line/point sink, Strack-type interface, subcritical-critical flow

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